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Unifying Principles for Sudden Transitions in All Systems

Jamshid Ghaboussi

*Department of Civil and Environmental Engineering,
University of Illinois at Urbana-Champaign, Urbana, Illinois*

Abstract

All physical, natural, biological and socio-economic systems – also referred to as complex systems - have system-level properties that result from the interactions between their components. In most cases it is not possible to determine the complete system-level properties with the current state of our knowledge. Where there are system-level properties, the uncoupled form of those properties is the eigen-system consisting of system eigenvalues and eigenfunctions, even though at the present we are not able to determine them through modelling or observation.

All systems operate in equilibrium states; small perturbations cause small changes. While these systems normally undergo gradual changes in their system-level properties, they can also undergo sudden transitions to new equilibrium states. New insights into these important transitions are proposed in this paper. In some mechanical systems transition occur when the smallest system eigenvalue goes to zero. It is proposed that the same principles apply to all systems. Transitions in all systems occur when at least one system eigenvalue goes to zero. Generalization of these principles to all systems will encourage new ways of thinking about systems and will suggest new research directions in studying these important major transitions, potentially leading to reliable methods for predicting their onset.

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1. Introduction

Although there is no universally accepted definition of self-regulating and self-organizing systems without any central control, it is generally agreed that these systems are made up of many interacting components, parts, elements or agents. These components have their own properties. The system properties are the result of the component properties and their interactions. These concepts have been present and generally accepted since they were first introduced by Bertalanffy in 1934 [1, 2]. The

universe is made up of systems such as physical, mechanical, natural, biological and socio-economic. The general definition of systems also includes complex systems, adaptive systems and system of systems such as eco-systems, the human immune system, the human brain, soft tissues, cells, economy, society, etc. Most of these systems are system of systems. For example, the components of eco-systems are species that are themselves systems. The general definition of systems also includes man-made systems such as buildings, bridges, utility systems and many mechanical systems. In general, systems are collections of many components with self-organizing interactions without any central control.

All systems have system properties even if we can not measure and model them. In most cases our current state of knowledge does not allow us to determine the system properties. Normally, systems are in equilibrium states. They remain in equilibrium states while they undergo gradual changes in the system properties. The systems can also reach tipping points that may lead to sudden transitions from one equilibrium state to another. These transitions often have far reaching consequences. They have been the subject of many studies in fields such as, ecosystems, environmental systems, stock markets and social systems (for example see references [3-10]). Many transitions do also occur in the human body leading to various diseases, even though they are not considered as transitions. The importance of gaining insight into these transitions is well recognized and can not be overemphasized. The understanding of the fundamentals governing these transitions is likely to lead to the development of reliable methods of forecasting them. A set of unifying principles governing these important transitions is proposed in this paper.

One method of forecasting the system transitions is through modelling and computational simulation. Properties of some physical and mechanical systems are well understood and well established mathematical and computational models describing their behaviour do exist. Other systems, especially complex systems such as socio-economic systems, eco-systems and many biological systems are difficult, if not impossible, to model accurately with the current state of our knowledge. The primary difficulty is in modelling of the components of the systems and their interactions. The components of most systems can not be tested or observed in isolation and methods for in-situ determination of their properties and interactions currently do not exist. Recently, a possible methodology for accomplishing this has been proposed [11], but it is at early stages of development and is not yet fully proven.

Although modelling of most actual systems is not currently possible, various methods have been used to model hypothetical systems in order to study and gain insight into their behaviour. These modelling methods have used computational intelligence methods, machine learning, fuzzy logic, cellular automata and agent based methods, (for example see references [12 - 15]).

Gaining insight into the behavior of actual systems, rather than hypothetical systems, is extremely important and is more likely to lead to the development of methods for forecasting the transitions. For example, accurate models of the economy could lead to reliable forecasting of recessions. Similarly, accurate models of the biological systems could lead to better system-wide understanding of diseases and methods for predicting their onset. In-depth understanding of the behavior of actual systems can be accomplished either through reliable modeling and computational simulation of the actual system, or through other observational methods that may lead to understanding and monitoring of certain important aspects of the system behavior. Ideas proposed in this paper are likely to lead to new approaches for achieving the latter. These ideas are based on transferring some aspects of our understanding of the mechanical systems to other more complex systems. It is proposed that some fundamental properties of systems are present in all the systems.

2. Equilibrium States and Transitions to New Equilibrium States

All systems exist in “*equilibrium states*”, defined here as states where small perturbations (external or internal) cause small changes. Systems undergo changes and their properties also change; the equilibrium states undergo gradual changes while the system remains in equilibrium state, meaning that the property

of small perturbations causing small changes remains. An example of systems is buildings, although they are not normally considered as systems. The components of the load carrying structures within the buildings are girders, beams, columns, walls, floors, foundations, etc. The properties of the buildings define their behaviour and how they respond to external forces, such as vertical loads, high wind forces and earthquakes. The properties of buildings are the result of the properties of their components such as beams, columns, etc and the interaction between those components. If the building is in equilibrium state, small forces will cause small displacements. Another example is a socio-economic system; its components are individuals, corporations, businesses, organizations, government agencies, etc. Socio-economic systems operate in equilibrium states; small perturbations such as small changes in interest rates and fuel prices (external perturbations) or small changes in the population demographics (internal perturbation) cause small changes in the system. The system remains in equilibrium state.

While the changes in the equilibrium states of the systems are normally gradual and small, all systems are also capable of undergoing major, and often sudden, transitions from one equilibrium state to another. When a system reaches a critical tipping point - a state at which a major transition can occur - small internal or external perturbations can cause large changes in the system, causing it to transition from one equilibrium state to another.

In the example of the building in its normal equilibrium state, small perturbations (such as a small external force or a small reduction in the size of the columns and beams) would result in small changes. However, if the building's loads are increased or the size of some critical columns and beams are reduced, it will reach a tipping point where small perturbations can cause large changes and the structure can collapse. Normally, the collapse of a building is not considered a transition from equilibrium state to another. However, if buildings are considered to be systems then it is logical to also consider that the collapse of a building is a transition from its normal and desirable equilibrium state to the collapsed and undesirable equilibrium state. There are many similar examples of transitions that occur in systems that are not traditionally considered as system.

Similar observations can be made in the example of the socio-economic systems. The economy normally operates in an equilibrium state where small perturbations cause small changes, until the accumulation of changes make the system so sensitive that small perturbations can cause major changes and a sudden transition to a new equilibrium state occurs. The US and world economies experienced a similar tipping point situation in 2008, where small perturbations (that always exist as a result of normal system fluctuations) caused major changes. Another recent example is the case of the societies in the North African and Middle Eastern countries which had reached tipping point states where small perturbations, such as the self-immolation incident in Tunisia, caused major transitions toward new equilibrium states.

There are many examples of these transitions in physical, natural and biological systems. It is well known that natural eco-systems that normally operate in equilibrium states can undergo major irreversible transitions to new equilibrium states when major changes occur in the population of a species within the system. Another example of the transitions in the biological systems is when healthy cells become cancerous. This can be considered as a transition from the healthy equilibrium state to the cancerous equilibrium state. Healthy cells are operating in equilibrium states where small perturbations are not causing major changes. Similarly, cancerous cells are also in equilibrium states and they can resist small perturbations.

3. Transition to New Equilibrium States in Mechanical Systems

The properties of mechanical systems, such as building structures, are well understood in the engineering community and it is possible to develop reasonably accurate and reliable computational models describing their behaviour. The structural systems are designed to remain stable (in equilibrium state) without undergoing large displacements and deformations.. Transitions to new equilibrium states

can occur when the structural system becomes capable of undergoing large displacements without requiring any additional energy. This occurs either when the structural system becomes unstable or when it loses its support and can undergo what is known in the engineering community as *rigid-body-motion*. In both cases at least one of the eigenvalues of the structural system becomes zero. The eigenvalues and the eigenvectors of the structural systems can be determined from their computational models. We can also define the eigenvalues and the eigenvectors as the *uncoupled properties* of the system; all the possible responses of the system can be described as a linear combination of the system eigenvectors. Normally the response of a structure is the linear combination of the few lowest eigenvectors. The eigenvalues of structural systems are the energy required to deform the structure along the corresponding eigenvector. When an eigenvalue become zero no additional energy is required for the structure to start moving along the corresponding eigenvector and this normally initiates the transition to a new equilibrium state. We will further describe these transitions through some examples.

Consider the example of a simple column fixed at the base and subjected to axial load at the top. The column normally operates in equilibrium state; a small lateral force will produce a small lateral displacement. If the axial load is increased, at some point the small lateral force will produce large lateral displacements. This is when the column buckles; it transitions from its original equilibrium state to the buckled equilibrium state. At this point one of the eigenvalues of the column has become zero. Another important observation is that the initial stage of the transition to the buckled state occurs along the eigenvector (or eigenfunction) of the zero eigenvalue.

Next we consider the example of buildings that normally operate in equilibrium states when small perturbations, such as a lateral force caused by wind, causes small displacements. If a building is continually loaded, its lowest eigenvalues keep decreasing. When one of the eigenvalues of the building structure becomes zero, the building has become unstable and small perturbations, such as a lateral force, can cause major changes and cause the transition to the collapsed equilibrium state. It is interesting that we do not normally consider the collapse of a building a transition to a new equilibrium state but that is exactly what it is.

Buckling or instability in the structural systems is well understood and they are always associated with at least one eigenvalue of the system going to zero [16]. As explained earlier, the eigenvalues of structural systems are the energy required to deform the structure along the corresponding eigenvector. The zero eigenvalue means that the motion along its eigenvector requires no additional energy and it forms the initial path of the transition to the buckled or collapsed equilibrium state. Once the transition starts, the system changes and its properties are no longer the same as the original structure. The lowest eigenvalues of the changing systems remain zero through the transition until its completion.

Another possibility for zero eigenvalues in structural systems is when they can undergo rigid-body-motion, motion that requires no new force and causes no deformations in the structure. For example, if the columns on one floor of a building are suddenly removed, the building can undergo rigid-body-motion leading to collapse, or transition to a new equilibrium state. This occurs during the demolition of some high-rise buildings when the columns of the first floor are suddenly destroyed with explosives. The remaining structure suddenly has a zero eigenvalue and it can move down as a rigid body and this initiates the transition to the collapsed equilibrium state.

When an eigenvalue of a structural system becomes zero it has either become unstable or the structural system can undergo rigid-body-motion. In both cases it transitions from one equilibrium state to another. In this context, another definition of the equilibrium state is when all the eigenvalues of the system are non-zero.

4. Transition to New Equilibrium States in All Systems

It is self evident that all the systems have system-level properties. Although system-level properties can be accurately determined and modelled in some systems, such as structural systems, they are

extremely difficult and often impossible to model in many other systems. However, the system level properties do exist in all the systems, even if we can not determine them at the present. Where there are system level properties, the uncoupled form of those properties are the eigenvalues and eigenfunctions of the system. All the system responses to external stimuli are the linear combination of its eigenfunctions. It is proposed here that all systems have eigenvalues and eigenfunctions, even if we do not know how to determine them at the present time.

Proposition 1: *All the systems, including complex systems, have system level properties and the uncoupled properties of the systems are their eigenvalues and eigenvectors (eigenfunctions).*

The system level properties can change as the systems continually undergo changes due to external and internal perturbations. These changes also cause changes in the eigenvalues and eigenvectors (eigenfunctions) of the systems. As long as all the eigenvalues of the systems are non-zero the systems are in equilibrium states; small perturbations cause small changes.

Proposition 2: *All the systems have equilibrium states where small perturbations result in small changes in system properties. All the eigenvalues of a system are non-zero when the system is in equilibrium state.*

When the eigenvalues of a system undergo changes, under some conditions at least one of the eigenvalues of the system can approach zero. At that point small perturbations can cause major changes; the system undergoes transition from one equilibrium state to another.

Proposition 3: *All systems can undergo major transitions from one equilibrium state to another. When at least one of the eigenvalues of a system goes to zero, small perturbations can result in major changes in the system; the system is ready for a transition to a new equilibrium state.*

Proposition 4: *The initial path of the transitions to new equilibrium states is along a linear combination of the eigenvectors (eigenfunctions) of the zero eigenvalues.*

These four propositions form the basis of a unifying principle for all physical, mechanical, natural, biological, socio-economic systems, including all complex systems, adaptive systems and system of systems. They provide a new way of thinking about all systems, especially about the all important sudden transitions that cause these systems to transition from one equilibrium state to another.

These propositions also point to new research directions for developing methods of determining the eigenvalues of most systems, or developing methods for monitoring the changes in lower eigenvalues of the systems from observational data on the system behaviour. Rapid decline in the lower eigenvalues may point to the increased likelihood of approaching sudden transitions to new equilibrium states. This will be very useful in socio-economic systems, eco-systems and bio-medical systems. If monitoring of the changes in the lower eigenvalues of the economy were possible, it would have provided one more indication of the approaching financial crisis and recession in 2008. In the field of bio-medicine, new research may lead to the development of monitoring methods for detecting the approach of certain diseases.

5. Conclusions

It is proposed through four propositions that all systems, including complex systems, have system properties and the uncoupled form of the system properties are their eigenvalues and eigenvectors (eigenfunctions). All systems operate in equilibrium states when all the eigenvalues of the system are non-zero. When systems are in equilibrium states, small external or internal perturbations or stimuli cause small changes in the system. All systems undergo changes while they remain in equilibrium states and their system properties and their eigen-systems also change. All systems are capable of undergoing sudden transitions from one equilibrium state to another. When at least one of the eigenvalues of the system approaches zero, the system is ready for the transition to a new equilibrium state. In these states

small perturbations can cause large changes, initiating the transition to a new equilibrium state. The initial path of the transition to a new equilibrium state is along the eigenvector (eigenfunction) of the zero eigenvalue.

These propositions provide a unifying principle for the sudden transitions that can occur in all systems. They also point to new research directions in monitoring the lowest eigenvalues of the systems, leading to methodology for predicting the onset of the major transitions to new equilibrium states.

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